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6. AUTHOR(S)
F.E.C. CULICK

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CALIFORNIA INSTITUTION OF TECHNOLOGY
OFFICE OF SPONSORED RESEARCH
1200 EAST CALIFORNIA BLVD.
PASADENA, CA 91125

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13. ABSTRACT *(Maximum 200 words)*

This report is the final report of a three-year program with a 9-month NCE comprising both theoretical and experimental work on problems arising with unsteady combustion of gases intended for use in propulsion systems. The experimental work comprises two main efforts: (I) measurements providing the basis for inferring the local response function of a reacting mixture: and, (2) accurate determination of the behavior of a Rijkc tube. Part (1) is bascd on greatly extended equipment purchased before this program began, with a DURJP grant from AFOSR. The basic part of the facility is apparatus for PLIF (planar laser induced fluorescence) measurements of a combustion system exposed to a sinusoidally varying pressure field, p', with q ', the local fluctuation of heat release, appropriate ineasuremcnts allow inference of q'/p', proportional to the response function and q'p', the integral over volume is proportional to the Rayleigh factor. In principal, knowledge of these two quantities permits deduction of all important dynamical properties of a dynamical system in the second part of this program, a particularly simple combustion system, the Rijke tube, has been investigated. We believe that we have produced the most accurate data ever obtained to confirm quantitatively much of basic behavior of the device. This result does much to solidify the basis for theoretically and practically treating the dynamics of time-dependent combustion systems.

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EXPERIMENT AND THEORY FOR ACTIVE CONTROL OF NONLINEAR DYNAMICS IN COMPRESSION/COMBUSTION SYSTEMS

AFOSR Grant Number: F49620-99-1-0118

1 February 1999 – 31 December 2002

Principal Investigator

F.E.C. Culick

California Institute of Technology
Phone: (626) 395-4783
Fax: (626) 395-8469
Email: fecfly@caltech.edu

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Abstract

This report is the final report of a three-year program with a 9-month NCE comprising both theoretical and experimental work on problems arising with unsteady combustion of gases intended for use in propulsion systems. The experimental work comprises two main efforts: (1) measurements providing the basis for inferring the local response function of a reacting mixture; and, (2) accurate determination of the behavior of a Rijke tube. Part (1) is based on greatly extended equipment purchased before this program began, with a DURIP grant from AFOSR. The basic part of the facility is apparatus for PLIF (planar laser induced fluorescence) measurements of a combustion system exposed to a sinusoidally varying pressure field, p', with q', the local fluctuation of heat release, appropriate measurements allow inference of q'/p', proportional to the response function and $q'p'$, the integral over volume is proportional to the Rayleigh factor. In principal, knowledge of these two quantities permits deduction of all important dynamical properties of a dynamical system. In the second part of this program, a particularly simple combustion system, the Rijke tube, has been investigated. We believe that we have produced the most accurate data ever obtained to confirm quantitatively much of basic behavior of the device. This result does much to solidify the basis for theoretically and practically treating the dynamics of time-dependent combustion systems.

1 Research Objectives

The overall technical objective of this program is analysis and experimental investigation of nonlinear unsteady motions and other fundamental problems relating to active control of combustor dynamics, in particular combustion instabilities. It is an educational objective that all students involved in this program will gain experience in both experimental and analytical work. Moreover, they become well-versed in the fields of combustion and controls.

When this program started, we anticipated using a new continuous-flow facility operating at pressures of up to four atmospheres. However, it was to be a gift from a small commercial firm which encountered financial difficulties. The apparatus became unavailable. Therefore we revised our plan and designed and built a new device, a vertical atmospheric tube in which an oscillating field is driven by loudspeakers set in branches at the top. A combustion device

producing an initially unmixed flame is set near the bottom. Much of this program has been devoted to measuring properties of species produced in the flame using PLIF (planar laser induced fluorescence). The PLIF system was constructed with a DURIP grant from AFOSR. With this arrangement and the use of the PLIF system, it became possible to make the first "point" measurements of heat release associated with an oscillatory pressure field. The values are actually obtained for a volume of 0.05 mm, not a true "point." If developed further, this method has promise to give results for the sources causing combustion instabilities. Such results would not depend on the shape of the chamber.

We have designed and constructed a similar apparatus for operation up to five atmospheres, the limit of the usual methods based on PLIF. That device has been built but is not yet operating.

A second, separate effort in this program has been concerned with data taken with a more traditional form of electrically driven Rijke tube. The initial purpose was to obtain, at last, truly accurate data for the stability boundary of the device. We succeeded, but the work went much further, producing accurate calculations of stability while accounting for all relevant physical phenomena.

Some years ago the Principal Investigator published a simple model for vortex shedding in a dump combustor. In the present program that model (two-dimensional) was made more explicit and used as the basis for quantitative predictions. It appears that this approach holds considerable promise for extension to three-dimensions and application to full-scale motors, including gas turbines, ramjets and afterburners.

Finally, this program has been extraordinarily important in providing support as well as material for the Principal Investigator's short course on combustor dynamics. See the website www.its.caltech.edu/~culick

2 Introduction

This program was motivated by the variety of important problems arising in unsteady reacting flows present in propulsion systems. Mainly we have been concerned with situations presented with combustion instabilities. Experimentally our attention has especially been directed to measurement of the response function for a gaseous mixture in a general way, that is, unrestricted by the geometry of the combustor. One is therefore driven, almost inevitably, to use

methods based on lasers. We have been developing a technique based on PLIF, planar laser induced fluorescence. That this is a general and productive point of view may be seen in the following way.

A combustion instability is observed normally as oscillations in a combustion chamber. The oscillations are present because of a close connection between the dynamical behavior of the combustion processes—especially the associated energy release—and the properties of the unsteady flow, particularly the pressure, temperatures, velocity and mixture ratio. Analytical and theoretical work has been to a large extent devoted to this close connection and its consequences. In order to understand the matter and formulate the connection in a useful and productive fashion, simple models have been used. These models have, practically without exception, been based on oversimplified representations, usually of the behavior of flame sheets in gaseous and liquid-fueled systems. There is generally a desperate need for models of unsteady combustion processes distributed in realistic chambers. The main goal of our numerical simulations and of the experimental work justifying this proposal is to acquire the basic information necessary as the basis for constructing such models.

So far as we know, our experimental research in this area is unique, not only in respect to the content and purpose, but also in the immediate connection with modeling and theory. We know of no other research group attempting to measure directly the dynamics of flames and distributed combustion processes for the purposes we have in our program. The results are fundamental contributions to the field of reacting flows generally. In respect to practical applications, experimental and theoretical works on active control combustor dynamics currently tend to ignore the problem of determining the details of the mechanisms of instabilities and concentrate on trial and error *ad hoc* efforts to control the dynamics. The results are therefore almost always limited to the particular systems studied.

This program is, we believe, the first devoted to determining quantitatively a fundamental contribution to practically all unsteady motions in combustors: the dynamical response of the rate at which energy is released locally by chemical reactions.

What we mean by 'dynamical response' and why it is crucially important to the dynamics of combustors is best seen by examining a formal representation of Rayleigh's criterion:

$$\Delta E = K\int dV\left[\frac{1}{T}\int_{t}^{t+T}\dot{Q}_r' p' dt\right] \tag{1}$$

Here $\dot{Q}_r'$ stands for the fluctuation of the combustion energy release rate per unit volume that is **in phase** with the pressure fluctuation; K is a constant; the inner integral is over one period τ of the oscillation; and the outer integral ranges over the volume of the combustor. Then ΔE represents the total energy transferred from the chemical reaction to the oscillating field in one period of the oscillation. With the formula (1) Rayleigh's criterion states that if ΔE is positive, combustion processes tend to encourage the system to be unstable. Thus the criterion is equivalent to the condition one finds for linear instability of a combustion system (Culick 1975, 2001). Culick (1987) has discussed a general form of (1) including all relevant processes and applicable to nonlinear behavior, showing its relation to the general principles of stability.

Since data were first reported by Stirling and Zukoski (1991), a number of groups have reported confirmation of (1); the most recent results are those obtained by Kappei *et al.* (2000). In both of those cases, and others we are aware of, fluctuations of energy release were detected by measuring the radiation from CH, an intermediate species in hydrocarbon reactions. The radiation is typically collected through a slit or a window and measured with a photomultiplier. Thus the fluctuation of energy release is found for an extended region of the field.

The primary objective of the work reported here has been to determine local values of the fluctuations of energy release rate, and especially the dependence on instantaneous local values of the variables defining the field: pressure, temperature, mixture ratio of the reactants, and velocity. This objective has been accomplished by applying, to simple flow configurations, a combination of two experimental methods: planar laser induced fluorescence (PLIF) and a probe (the 'Dibble probe') for measuring initial concentrations of unreacted gaseous fuel (CH_4 here) to give spatially resolved and time-accurate results for $\dot{Q}_r'$ up to 55Hz. So far as we know, those are the first such results.

Accomplishment of the objective stated in the previous paragraph will provide the basis for constructing a model (or models) of the source term $\dot{Q}_r'$ in the integral (1), a primary cause of instabilities in combustors. By 'primary cause' we mean here that $\dot{Q}_r'$ (actually its time derivative) appears as a source of unsteady motions in the equations for flow in a combustor and therefore can be given a direct physical interpretation for a particular problem being considered. Accurate representation of $\dot{Q}_r'$ is essential to understanding the presence of combustion

instabilities and, accordingly, how they can be either avoided or eliminated in practical systems. Quite generally $\dot{Q}'_r$ must be known to analyze any problem of combustor dynamics.

2.1 Combustor Dynamics, Combustion Dynamics and Combustion Instabilities

Almost all[1] combustion chambers intended for propulsion or power generation systems are designed for steady operation, subject at most to relatively slow changes. The reacting flow in a combustor is defined entirely by the state of the reactants entering the chamber and the geometry, including the injector design. The practical and research problems we face here are related essentially to stability of the flow field. If the flow field is unstable, then the system exhibits unsteadiness i.e., in a general sense, dynamical behavior.

The dynamics of a combustor arise primarily from two sorts of instabilities: (1) instability of the steady flow field itself; and (2) instability of the intrinsic or acoustical dynamics of the chamber. Instability (1) of the steady flow leads to turbulence and formation of coherent vortex shedding associated with unstable shear layers. Both types of unsteady disturbances affect the distribution of chemical activity in the chamber. It is the instability (2) of the acoustical motions, a basic property of compressible flow in any chamber, that leads to the dynamical motions called combustion instabilities.

In practical combustors, the existence of combustion instabilities cannot be considered completely independently of instabilities of the basic flow. The flows are always turbulent, affecting the rate of energy release due to combustion and the form of the macroscopic flow field, including shear layers, recirculation zones and large-scale vortices, is a primary influence on the distribution of energy release rate, $\dot{Q}$. Then the fluctuation $\dot{Q}'$ of $\dot{Q}$ appears as the primary source of the unsteady acoustic field, as discussed in connection with the formula (1).

A basic premise of our work is that the *local* values of $\dot{Q}$ and $\dot{Q}'$ are in fact determined by *local* values of the variables characterizing the flow field. Therefore *if* that fundamental dependence can be found for a particular system of reactants, a basis exists for understanding the causes of combustion instabilities in any combustor using the same reactants and primarily due to $\dot{Q}'$. That idealized flow of information is shown schematically in Figure 1. Current work

[1] The chief exceptions are pulse combustors, not the object of the work discussed here.

includes the items represented by the yellow shaded blocks. As a result of work at Caltech, the essentials of the methods indicated in the blue block already exist for the most part. The procedures are general, capable of describing linear and nonlinear behavior for any geometry, but to obtain specific results, models of certain dominant processes are required. **Constructing those models for a broad range of realistic conditions is an ultimate objective of our long-term program including the work described here.** The experimental information required to construct those models is fundamental to research on reacting flows generally. Hence the methods we develop will have very broad applications.

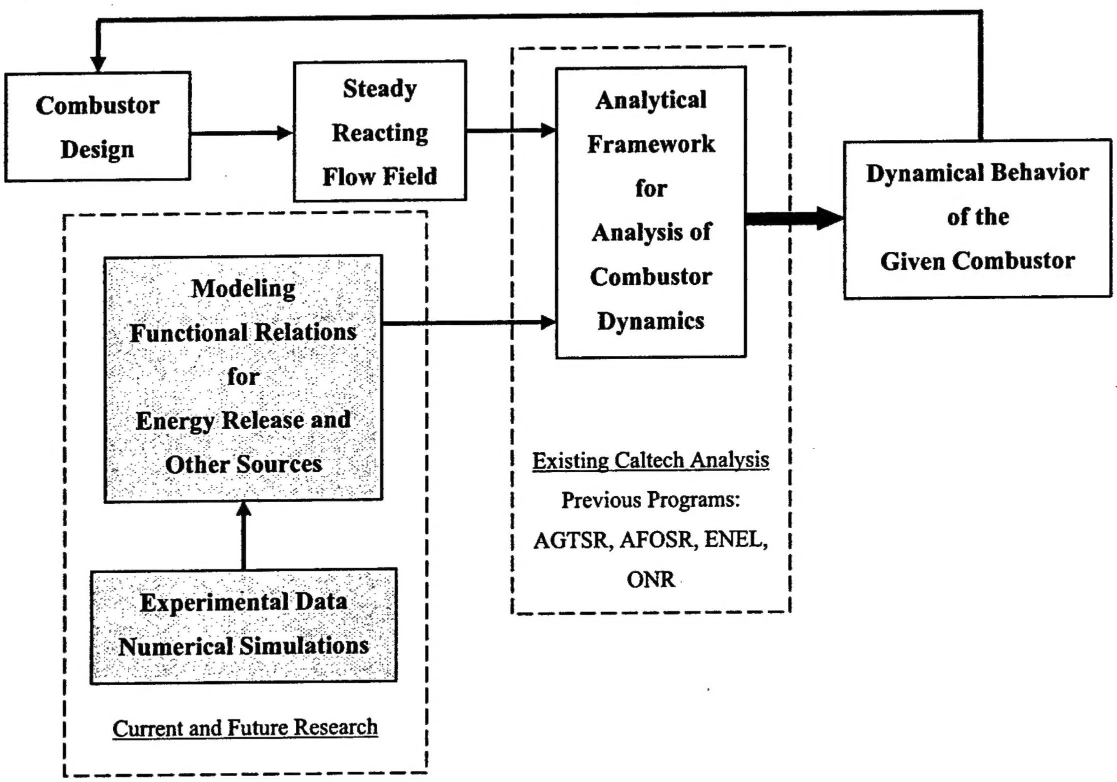

Figure 1. Idealized Flow of Information.

Rayleigh's criterion in the form (1) illustrates the central importance of the fluctuation of heat addition in combustor dynamics. The general theory, from which (1) can be derived, provides the framework for investigating the dynamics of any combustor. In Appendix B we include an elementary analysis of the Rijke tube, the simplest example of thermoacoustic or combustion instabilities. That analysis shows that due to the fluctuations of energy release the amplitude $\eta(t)$ of the fundamental acoustic mode has the exponential time dependence,

$$\eta(t) \sim e^{\alpha t} \tag{2}$$

Thus if α is positive, the amplitude grows, i.e. the mode is unstable.

The analysis shows also that the growth (or decay) constant is proportional to the real part of the combustion response to pressure fluctuation,

$$\alpha \sim R_p^{(r)} \tag{3}$$

where the complete complex response function is proportional to the ratio of the fluctuation of energy release to the local fluctuation of pressure,

$$R_p^{(r)} \sim \frac{\dot{Q}_r'}{p'} \tag{4}$$

That R_p is complex means that in general an oscillation of energy release has a phase lead or lag relative to the pressure oscillation. According to (3) and (4), if the energy fluctuation has a part in phase with p (so $\dot{Q}_r'$ is positive) $R_p^{(r)}$ and therefore α are positive. Hence (2) shows that the amplitude of the mode grows in time and is unstable. That $\dot{Q}_r'$ appears both here and in the formula (1) for Rayleigh's criterion suggests (what is true) the equivalence of Rayleigh's criterion and the principle of linear stability (Culick 1987, 1992, 2001).

The preceding is the simplest reasoning to show the fundamental role of energy fluctuations in causing combustion instabilities. Extension of the reasoning (see Appendix C) shows that the fluctuations are fundamental as well to applications of active control of combustion systems.

In the context of the dynamics of combustion systems, notably combustion instabilities and active control, fundamental combustion dynamics occupies a central position. Knowledge of the combustion response function is essential to understanding and modifying the dynamical behavior of a combustor. Several combustion response functions exist, representing the fluctuations of the rate at which energy is released in chemical reactions in response to changes of pressure, velocity, temperature, and mixture ratio. For use in applications to active feedback control, those response functions are interpreted as transfer functions.

Response or transfer functions must be modeled in some way to carry out analysis of the dynamical behavior of a combustion system and to do a prior design of active control. Currently, in the absence of theoretical and experimental information, simple *ad hoc* models are used in all applications. **The main purpose of the research described here is to continue developing, extending and improving methods for inferring response functions by using data obtained**

with laser-based diagnostics, principally PLIF and, eventually PIV. Roughly, the strategy is the following.

The basis for the method generally is that the chemical process of producing energy is accompanied by generation of short-lived species. For combustion of hydrocarbon fuels, the radicals OH and CH are especially significant because many previous works have established that to good approximation the rate $\dot{Q}_r$ at which energy is released by the combustion processes locally is proportional to the concentrations of those species. How good the approximation is depends on, among other factors, the local macroscopic flow and mixing rates, and the collisional destruction of the radicals.

Hence, to some (often good) approximation, measurement of the concentrations of OH and CH can be related to the rate of energy production. This conclusion holds true under unsteady conditions, to some approximation, if the characteristic time of the unsteadiness (here the period of impressed oscillations) is long compared to the measurement time, which in turn is short compared with the time in which the concentrations change significantly.

The first purpose of this research is to impress oscillations of pressure and measure the fluctuations of one of the radicals in question, OH being the simplest to observe. Then by assumption, the average and fluctuating heat release ratios ($\bar{\dot{Q}}$ and $\dot{Q}'$) are proportional to the average and fluctuating values of the OH concentration (denoted $\overline{[\mathrm{OH}]}$ and $[\mathrm{OH}]'$, respectively). So by assumption,

$$\begin{aligned} \bar{\dot{Q}} &= \bar{K}\,\overline{[\mathrm{OH}]} \\ \dot{Q}' &= K'[\mathrm{OH}]' \end{aligned} \qquad (5)\ \text{a,b}$$

If $\bar{K}$ and K' have the same values, then we can write,

$$\frac{\dot{Q}'}{\bar{\dot{Q}}} = \frac{[\mathrm{OH}]'}{\overline{[\mathrm{OH}]}} \qquad (6)$$

As in (4) above, a response function R_p is conveniently defined as fluctuation of the heat release rate to the value of the impressed pressure fluctuation,

$$R_p = \frac{\dot{Q}'}{p'} \quad \text{or in dimensionless form} \quad R_p = \frac{\dot{Q}'/\bar{\dot{Q}}}{p'/\bar{p}} \qquad (7)$$

Hence with (6), R_p is expressed as a ratio of measurable quantities:

$$R_p = \frac{[\mathrm{OH}]'/\overline{[\mathrm{OH}]}}{p'/\overline{p}} \tag{8}$$

where PLIF measurements give the numerator, and the denominator is obtained with pressure transducers. It is this ratio, R_p or $R_p^{(r)}$ which appears explicitly in analysis of the dynamics of combustion systems as described above.

2.2 Previous Work on Measuring Combustion Dynamics

Three methods have been used to gain data for combustion dynamics. In order of increasingly finer spatial resolution, they are:

(i) measurement of the transfer function for a combustion region by detecting transmission and reflection of acoustic waves (*e.g.,* Culick 2001);
(ii) chemiluminescence, measuring radiation from certain species participating in a combustion zone exposed to pressure oscillations; and
(iii) planar laser-induced fluorescence, measuring radiation induced by pulses of laser output incident on a combustion zone exposed to pressure oscillations.

We are using both the second and third methods. In fact one unforeseen result has been to clarify the serious limitations of chemiluminescence.

Put briefly, the method based on chemiluminescence involves observation of 'natural' radiation by using either (1) a photomultiplier tube (PMT) with a slit obscuring all but a portion of the combustion zone, giving resolution in two dimensions; or (2) a CCD camera capable of giving resolution in two dimensions. The great disadvantage of this method is that the radiation collected is emitted along the entire line of sight in the direction defined by the orientation of the PMT or camera. Since radiation may also be absorbed, the final intensity at the observation point does not in general represent only the activity of species produced in chemical reactions. Consequently, as we have shown (Pun *et al.* 2001) seriously misleading results are often obtained. Nevertheless, the method was the first to provide results for combustion dynamics (see Table 1) and has given useful contribution to understanding combustion instabilities.

Chemiluminescence of the CH radical, an excellent marker for the reaction zone, has been used by a number of researchers to study heat release in an unsteady flame. They can be categorized into two groups; measurements using a PMT with a slit obscuring a portion of the flame to obtain some spatial (typically axial) resolution (Sterling 1991; Chen *et al.* 1993; and

Kappei *et al.* 2000); and fully two-dimensional imaging using a CCD based camera (Broda *et al.* 1998; Kendrick *et al.* 1999; and Venkataraman *et al.* 1999). Of these works, only Chen *et al.* (1993) involved an acoustically forced flame, but used a PMT with a slit configuration that obtained only integrated one-dimensional information.

The first demonstration of 2D (planar) LIF of the hydroxyl radical in a flame was apparently performed by Dyer and Crosley (1982). This technique has been used to measure a variety of chemical species in unsteady reacting flows, including OH as a measure of the heat release (Cadou *et al.* 1991; and Shih *et al.* 1996), and NO seeded fuel to measure the temperature field (Cadou *et al.* 1998). A summary of these various works involving both chemiluminescence and PLIF is provided in Table 1, including the acoustic frequencies in the studies.

Most experimental work to characterize various combustor configurations has been done on naturally unstable systems (see Table 1). However, the results are specific to the combustors tested, and provide little insight to how a particular injector or burner design will behave in a different combustor. A study of the acoustic coupling between fuel injectors and an applied acoustic field has been carried out by Torger (1998), but only includes cold flow experiments. Work by Chen *et al.* (1993) with premixed flames was specifically designed to simulate solid rocket propellants. It contained one-dimensional spatial results and used only two forcing frequencies. The study by Cadou *et al.* (1998) was based on a specific 2D dump combustor configuration, and showed little response to non-resonant forcing. A more generalized body of work is required to provide industry with guidelines that will be useful in designing stable combustion systems.

	Chemiluminescence	PLIF
Naturally Unsteady	• Sterling and Zukoski (1991) (188 Hz) • Broda *et al.* (1998) (1750 Hz) • Kendrick *et al.* (1999) (235 Hz, 355 Hz) • Venkataraman *et al.* (1999) (490 Hz) • Kappei *et al.* (2000) (370–460 Hz)	• Cadou *et al.* (1991) (43 Hz) • Shih *et al.* (1996) (400 Hz) • Cadou *et al.* (1998) (328 Hz)
Acoustic Forcing	• Chen *et al.* (1993) (300 Hz, 400 Hz)	• Cadou *et al.* (1998) (360 Hz, 420 Hz)

Table 1. Previous work in oscillating flames.

In Section 4.1 we describe our apparatus for applying PLIF to an acoustically forced flame. Only Cadou (1998) previously used PLIF to investigate the dynamics of a forced combustion

system. Our experiments began (with his help) essentially where Dr. Cadou finished. Otherwise, as Table 1 shows, in previous applications of PLIF, to unsteady flames, results were obtained only for naturally oscillating systems. Cadou forced his system at only two frequencies. Only by forcing over a range of frequencies can one obtain data for transfer functions and truly informative results for the combustion dynamics.

2.3 Work at Caltech on Measuring Combustion Dynamics

Before 1995, the work by the Principal Investigator and his students on dynamics of combustion systems was devoted almost entirely to theory and analysis, supported chiefly by AFOSR and ONR. In parallel, from about 1981 to 1994, in separate programs funded by AFOSR and ONR, continuous research had been devoted to instabilities in dump combustors; vortex shedding and associated periodic combustion was the primary mechanism. In 1996–1997, the PI and three students carried out a small project successfully showing that hysteresis in a region adjacent to the stability boundary could be used as the basis for nonlinear active control to extend stable operation over a broader range of mixture ratio (Isella, Seywert, Culick and Zukoski 1997).

Analytical work at Caltech on active control of combustion systems was being supported by AFOSR, ONR, and, beginning in the mid-1990's, by the Department of Energy. Our emphasis was on the dynamics of combustors generally with attention paid to various classes of problems within that subject. For example, we have worked out the only analytical method for investigating active control of instabilities in the presence of noise (Seywert, Isella and Culick 2000). It became increasingly clear that for understanding both the laboratory results and the behavior observed in full-scale systems, the real obstacle was understanding the fundamental mechanisms; namely the dynamical conversion of chemical energy to the mechanical energy of unsteady motions in flames and on the reacting flows. That understanding can come only from detailed experimental results taken for sufficiently large frequencies covering the range of actual behavior, at least to 1000 Hz.

Our experimental work in this area began in 1998 with a DURIP Grant from AFOSR, with additional funding from Caltech, DOE and ENEL, the Italian Power Company. Those funds

allowed us to construct the PLIF system now in place, that produced the very promising results reported first in July 2000 and recently published (Pun, Palm and Culick, 2003).

3 Faculty and Postdoctoral Personnel

F.E.C. Culick joined the faculty of the California Institute of Technology after receiving his Ph.D. in Aeronautics and Astronautics from MIT in 1961. He is currently Richard L. and Dorothy M. Hayman Professor of Mechanical Engineering and Professor of Jet Propulsion. Dr. Culick's Ph.D. dissertation was on the subject of combustion instabilities in liquid rockets and much of his research since then has been concerned with problems of unsteady motions in combustion chambers generally. He began working on solid rocket combustion instabilities in 1965, and since 1979 has been addressing the problem in air-breathing systems, including combustors and afterburners for gas turbines. He currently teaches the introductory course on control of physical systems, and, on alternate years, an introductory course on combustion or a course on performance, stability, and feedback control of aircraft. Dr. Culick is a Fellow of the American Institute of Aeronautics and Astronautics and a member of the International Academy of Astronautics. In 1981, he received the AIAA Pendray Aerospace Literature Award and in 1988 the JANNAF Combustion Subcommittee Recognition Award. From January to June 1992, he was appointed Professor Associate at École Central in Paris. From 1977–1986, Dr. Culick was a member of the AGARD Propulsion and Energetics Panel, and resumed that position in 1994. He has been a consultant to all of the major US rocket companies as well as to various government organizations. For nine years until 1995 he was a member of the Technical Advisory Council for Sverdrup Technology, Inc., primarily concerned with operation of the propulsion test facilities at AEDC, Tullahoma, Tennessee. He is a member of the Pratt and Whitney Technical Advisory Council.

Dr. A. Ratner received his Bachelor of Science in Engineering and Applied Science from Caltech in June 1995. While there, he worked on a student run space experiment that proved both educational and yielded interesting data. From fall of 1995–2000 he was at the University of Michigan, Department of Aerospace Engineering, working for Prof. James Driscoll. At U of M, he was supported by a 5-year FXB Association Fellowship. This fellowship allowed him to

focus on research and academic work. He received Masters degrees in both Aerospace Engineering and Mathematics. In his research, he was first to perform PIV in several regimes: highly turbulent flames, high swirl flames, and supersonic flames. In addition, he performed simultaneous CH and OH PLIF, as well as simultaneous OH PLIF and PIV. He also obtained fuel efficiencies by gas sampling. Since October 2000, Dr. Ratner has been a Postdoctoral Research Scholar at Caltech. He is a member of the AIAA and the Combustion Institute.

Dr. W. Pun received his Bachelor of Applied Science in 1994 at the University of Toronto in the Engineering Science program. He went on to earn an M.S. in Mechanical Engineering at the California Institute of Technology in 1995, and his Ph.D. at the same institute in 2001. His thesis work involved measuring unstable acoustic instabilities due to combustion and other heat sources, using laser diagnostics such as PLIF. He successfully wrote a grant proposal valued at a quarter of a million dollars for laboratory equipment, and helped establish the JPC combustion laboratory at Caltech. He was a Guggenheim Postdoctoral Fellow at the California Institute of Technology and a Postdoctoral Scholar from June 2001 to July 2002.

4 Experimental Methods

The methodology adopted in the described here is based on making fundamental measurements that can the be related to practical issue. This approach produces results that are broadly applicable and not as dependent on the peculiarities of a specific experiment. The configurations employed were a horizontal electric Rijke tube and vertical, externally-driven Rijke tubes (also referred to as acoustic chambers). While the horizontal tube provides clear air flow vs. heating parameter separation, the vertical tubes provide a separation of the burner and flame acoustic characteristics from the chamber natural modes.

4.1 Acoustic Chamber

The experiments performed in the acoustic chamber (vertical tubes) are all focused on measuring the flame transfer function. Since the flame is dependent on multiple parameters (local pressure, local pressure time derivative, velocity field [local strain rate], local temperature,

local fuel mixture fraction, and even local flow reaction history), the experiments have assessed the transfer function between external pressure and local heat release, nitric oxide (NO) production, and fuel mixture fraction oscillations. The experiments involving NO production also demonstrated the lack of data concerning the accuracy and reliability of laser-induced fluorescence (LIF) measurement of NO concentrations for even moderately rich flames. A set of experiments were hence performed to assess NO LIF behavior in rich flames and associated computational models for NO concentration in such environments.

The first two experiments utilized the same burner, chamber, and data acquisition configuration. The first experiment involved planar LIF (PLIF) imaging of the hydroxyl radical (OH) as a marker for heat release. The technique employed examines the *variation* in OH concentration. The incremental change in OH concentration is closely tied to changes in local temperature which (for similar flame conditions) is directly tied to changes in the heat release rate. For small variations in OH concentration (~10%), the correlation to heat release rate is very good. The other important technical insight involves measurement and use of Raleigh index. A local Raleigh index is calculated at each point in the flow field, based on the local heat release rate and the known pressure variation. The pressure variation is designed to be spatially uniform, temporally oscillating (externally driven and verified by measurement near the flame zone). Raleigh index allows for the calculation of a general flame transfer function that is not chamber dependent. Results for each of the experiments are illustrated in Section 7.

The second experiment was very similar except for the measurement of NO vs. OH concentration. For NO, the correlation is direct and the results are reliable for any size of variation. The major errors for the NO transfer function are in the initial NO concentration measurements. This led to the set of experiments characterizing NO line shape behavior for typical rich flames. These experiments utilized NO PLIF techniques to acquire NO LIF type data. This is a benefit because careful measurement of flame, laser, and flow chemical conditions can be performed as data is acquired.

The last two experiments focused on the three-way coupling between pressure oscillation, fuel mixture fraction oscillation, and flame oscillation. This type of coupling makes it harder to define global transfer functions, so the initial focus has been to characterize the coupling between pressure oscillation and fuel mixing. This has been done two different ways. One experiment employed a fiber optic probe to perform infrared laser absorption. The laser

frequency was chosen such that it was only significantly absorbed by the fuel (methane). This probe method has the benefit of high temporal resolution (several kilohertz) and does not require a secondary fuel marker. The method yields data that accurately characterizes the local fuel oscillations across a wide spectrum. The second method provides images, but with poor absolute accuracy for fuel concentration. The second method uses acetone added to the fuel as a marker. Acetone PLIF is then performed and fuel concentration inferred from the acetone density. The structures seen in the acetone images are then correlated with the patterns found in the probe data. This method provides significantly better results than either method alone.

4.2 The Rijke Tube

A Rijke tube, representing a resonator with a mean flow and a concentrated heat source, is a convenient system for studying the fundamentals of thermoacoustics. At certain values of the main system parameters, a loud sound is generated through a process similar to that in real-world devices prone to thermoacoustic instability.

For the purpose of obtaining data on thermoacoustic instability for a broad range of system parameters, a horizontally oriented Rijke tube with mean flow provided by a fan and heat addition from a movable electrically heated gauze is employed. In this way, the three main system parameters — heater location, air flow rate, and power supplied — can be varied and controlled independently. To generate reproducible experimental results convenient for mathematical modeling, a quasi-steady data acquisition procedure was established: for fixed heater location and flow rate, the power is varied slowly enough to minimize the effects of unsteadiness of the temperature field in the system and of finite power increments that could cause nonlinear triggering of instability. To accomplish the experiment in reasonable time, an iterative method is employed to control the power supplied: large power increments are used for rough determination of the stability boundary, and fine steps near the boundary vicinity are utilized for accurate localization of the transition between stable and excited states.

A computerized data acquisition system allows to capture all important system variables in time-resolved manner. In addition, spatially resolved measurements of the temperature profile along the tube are obtained, since a non-uniformity of the temperature field plays a crucial role in initiating and sustaining thermoacoustic instabilities.

A hysteresis of the stability boundary, discovered in the Rijke tube in our work and characteristic to instabilities of combustion devices, depends on the history of the system, such as an order of parameter variation, and a noise level. The appropriate experimental procedures have been implemented to quantify the hysteresis phenomenon.

5 Experimental Apparatus

5.1 The PLIF System

The apparatus for the various experiments consists of several separate sub-systems. Chemiluminescence requires only a camera and the burner/chamber arrangement being employed for the experiment. Fluorescence measurements on the other hand require an intensified camera, lasers, and timing electronics. The PLIF system (Figure 1) is based on an Nd:YAG laser operating at 10 Hz, pumping a tunable dye laser, which in turn drives a mixer/doubler system. The Nd:YAG laser outputs 2 J/pulse at 1064 nm (IR) and is equipped with a secondary harmonic generation system to provide 1 J/pulse at 532 nm (green).

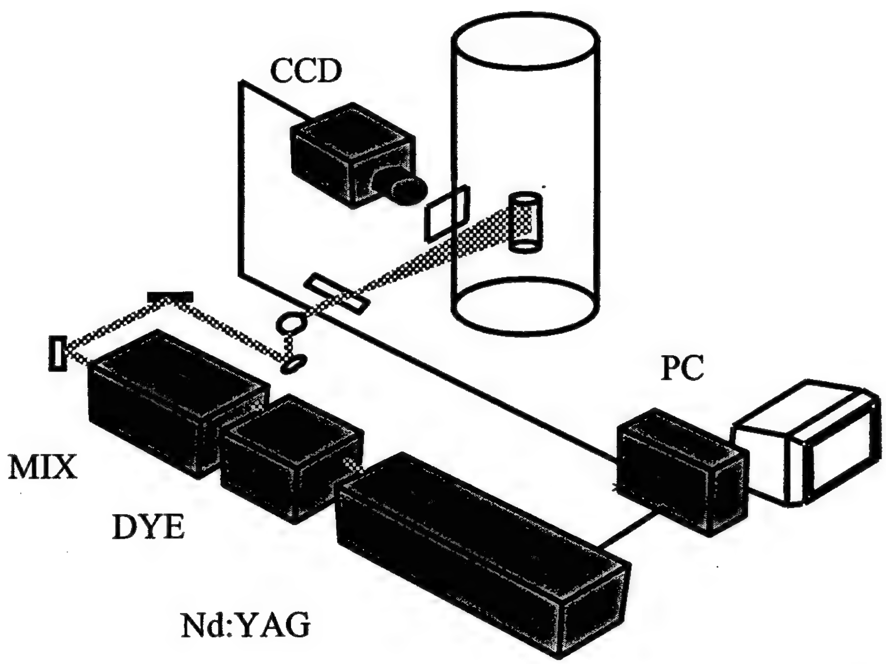

Figure 2. Schematic of the PLIF and CCD camera system, including a Nd:YAG laser, a tunable dye laser, and a frequency mixer/doubler.

The output of the Nd:YAG laser is used to pump a dye laser. The dye and laser excitation frequency are chosen based on the type of measurement to be performed. The output of the dye laser is then doubled to 282 nm (for OH), 280 (for acetone), or 287 and then mixed with the 1064 nm beam to produce approximately 226 nm (for NO). The final laser beam is narrowed using a plano-concave cylindrical lens and spread into a sheet by a plano-convex cylindrical lens. Final laser power varies between 5 mJ/pulse at 226 nm to 40 mJ/pulse at 282 nm of energy entering the test section to stimulate the species of interest (resulting in saturated fluorescence for most conditions). The resulting signal is detected by intensified CCD camera with 512x512 pixels resolution.

5.2 The Acoustic Chamber

Figures 3 and 4 show the atmospheric pressure acoustic chamber and the elevated (5 ATM) pressure acoustic chamber, respectively. The acoustic driving system is located in the upper portion of each chamber. The upper portion is made of a large tubular stainless steel section, in the shape of a cross, approximately 30.5 cm in diameter and 71 cm in height. For the atmospheric pressure acoustic chamber, the exhaust section is open to the atmosphere which provides an acoustically open exit condition. For the 5 ATM chamber, the exit is acoustically closed, and the exhaust is cooled before being released through pressure-controlling blow-down valves. The acoustic drivers in the upper portion of each chamber are protected for thermal extremes by a pair of air jet film cooling rings (to prevent heat failure of the drivers). The bottom acoustic condition is a closed-end condition for both chambers. Air is supplied at the bottom of each chamber. The total height of both chambers is approximately 1.78 m.

The acoustic drivers are 30.5 cm diameter subwoofers with a continuous power handling capacity of 400 W in the atmospheric chamber and 500W for the higher pressure chamber. The elevated pressure chamber employs 4 speakers (rather than 2) since the system is sealed and half of all speaker power is dissipated outside the main cavity. Both chambers have a power amplifier and a function generator that provide the power and signal to the acoustic drivers. The amplitude of the fundamental driving mode is actively controlled by custom-designed electronics, which measure the pressure in the acoustic chamber at the burner with a pressure transducer, and appropriately scale the power output of the speakers. The piezoelectric pressure

transducer is located at a height of 7.62 cm above the fuel spud, where the flame is stabilized in the burner. The signal from the transducer is notch-filtered to ensure the intended driving mode is correctly amplified or attenuated.

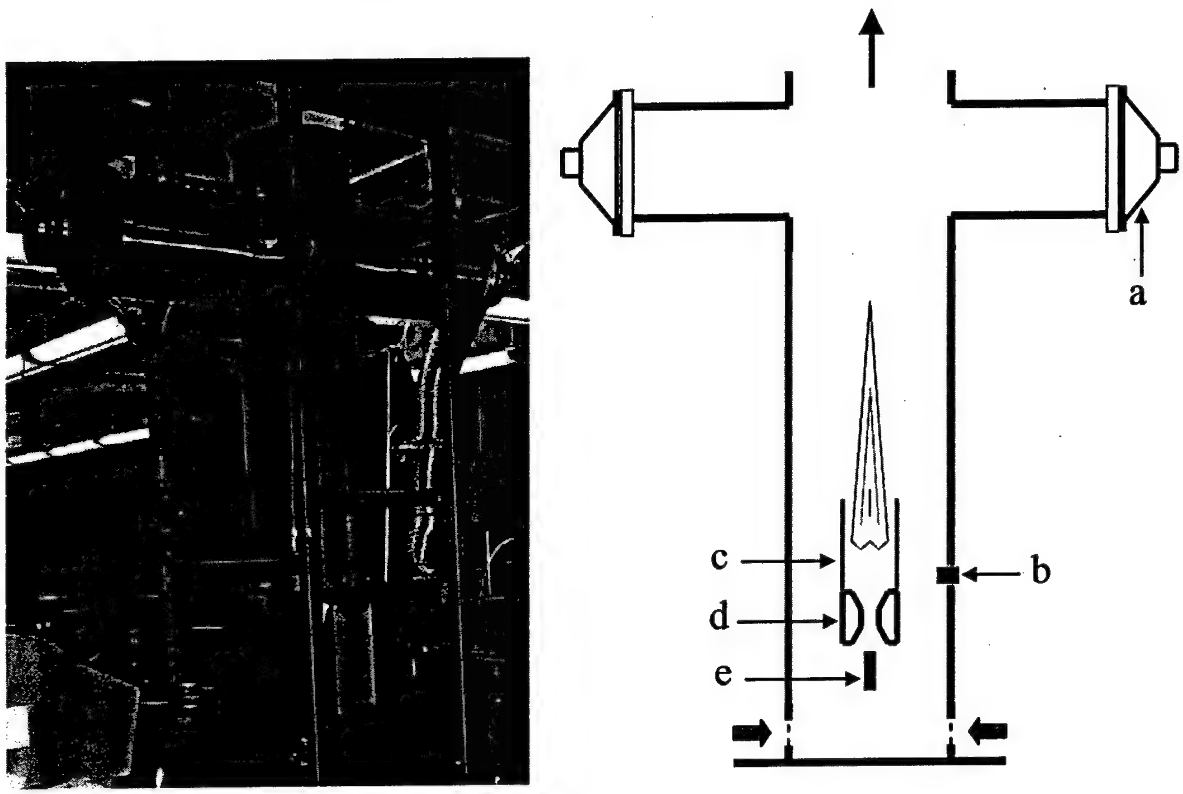

Figure 3. Photo (left) and schematic (right) of the atmospheric pressure acoustic chamber. The major components of the chamber are (a) acoustic driver, (b) pressure transducer, (c) fused-silica enclosure, (d) eductor block, and (e) fuel spud.

The burner used in these experiments consists of a fuel jet, an eductor block, and a flame enclosure (Figure 3). The fuel jet entrains air and partially premixes as it passes through the eductor. The flame is stabilized in the low velocity zone created as the flow exits the eductor, and expands into the fused-silica enclosure. The fuel jet is 0.428 cm in diameter and is located 2 cm below the eductor block. The eductor is 4.5 cm in height, has a 3.6 cm throat diameter, and is made of high temperature ceramic. The square-profile fused-silica enclosure mounts on top of the ceramic eductor. The enclosure is 11.43 cm in height and 5.72 cm in length on each side. The fuel jet is 50% methane premixed with 50% N_2 gas to increase the mass flow and produce a permanently blue flame. A blue flame (with no soot or soot precursors) is necessary because the black-body radiation from solid particles would over-whelm the weak signal that is being

measured for NO PLIF. The outlets for each gas are choked, in order to prevent disturbances from propagating upstream and affecting flow rates.

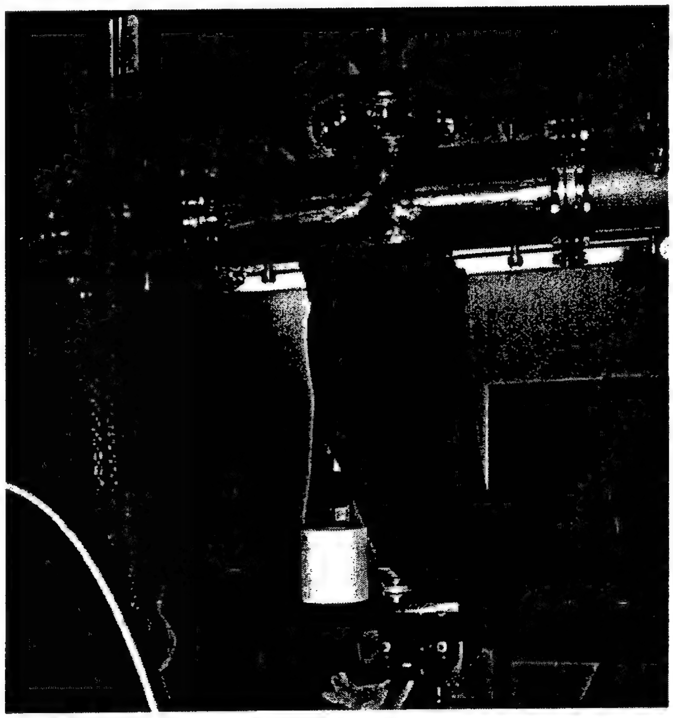

Figure 4. 5 ATM acoustic chamber. Lower portion of chamber covered with black cloth to reduce laser scatter. Chamber is symmetric across the vertical axis. Portion of the chamber is cut-off in this picture (right edge).

5.3 The Fiber Optic Probe

The other important piece of apparatus is the fiber optic probe (Figure 5). The probe assembly consists of a He-Ne laser which produces an infrared laser beam with wavelength of 3.39 micron. The laser beam is coupled into the sapphire optical fiber by a CaF spherical lens. The fiber passes into the probe and terminates at the edge of the measurement volume. The laser beam then shines across the volume, reflects of off a gold, front-surface mirror, and passes once more through the measurement volume. A receiving fiber is mounted next to the illuminating fiber, and takes in all incident light. This light then travels down the fiber to a solid state detector. The probe was inserted through the eductor block at the throat portion and traversed radially to sample the gas from centerline to the eductor wall.

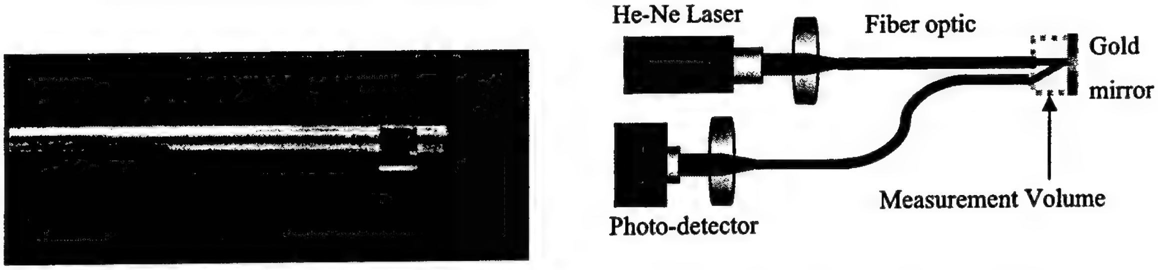

Figure 5. Photograph of probe (left) and schematic of probe laser system (right). The opening is approximately 5 mm per side and the probe is 6.3 mm in diameter.

5.4 The Rijke Tube

The structure of the experimental setup of the Rijke tube is shown in Figure 6. The horizontally placed square aluminum tube has 9.5 x 9.5 cm cross-section and length 1.0 m. The thickness of tube walls is 3 mm. Mean air flow is provided by a blower, which sucks air in the tube. The usage of a fan allows us to control a major system parameter, the mean flow rate, precisely and independently of the thermal power release, which is also regulated. To exclude the influence of natural convection on a mean flow rate, the horizontal orientation of the Rijke tube has been implemented. A damping chamber, located between the tube and the blower, is intended to prevent influence on the blower from tube acoustics. The chamber dimensions are 46 × 46 × 120 cm; its internal surface is covered by 1/2" pile carpet on 1/8" felt.

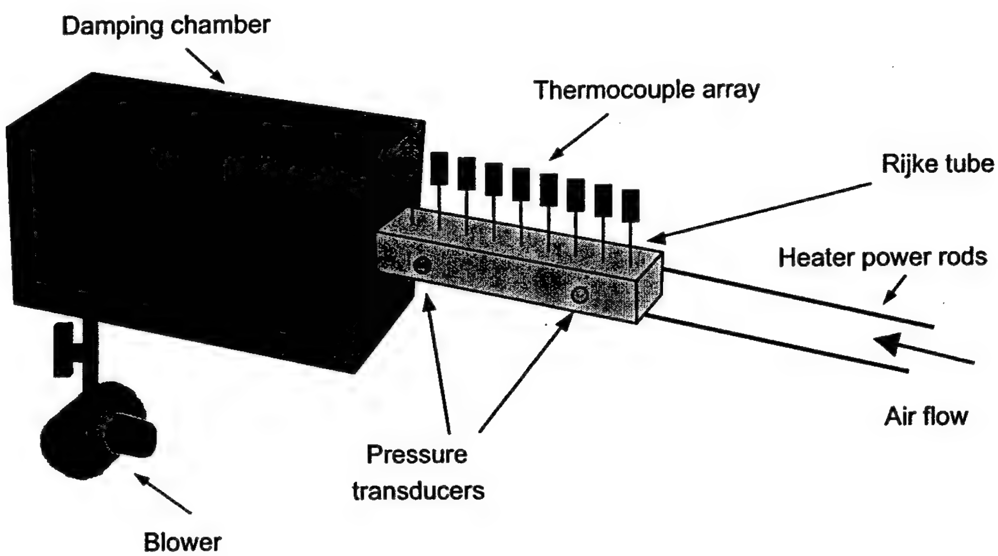

Figure 6. Experimental setup of the Rijke tube

Since the major mystery about Rijke oscillations is associated with the heat transfer at the heater, it is desirable to select a heater with the heat transfer process that can be analytically modeled as precisely as possible. This suggests an array of parallel cylinders, located far from each other. In order to consider a heater to be infinitely thin in mathematical modeling, the cylinder diameter must be small, hence, a set of wires could be used. To measure stability properties of a Rijke tube in wide ranges of system parameters, a heater should also repeatedly withstand high temperatures for long time intervals and be able to release large amounts of thermal power (over 1 KW in our system) without changes in geometry. Another desirable property is the ability to heat the air flow uniformly over a tube cross-section. A square-weave 40-mesh made from nichrome is a suitable trade-off between these requirements. It has proved to be a reliable heating element, and the heat transfer resembles that from cylinders, accounting for a flow blockage effect. A wire diameter in this grid is 0.01". The gauze is brazed to two strips of copper, which are suspended on a square frame made from macor in order to eliminate electric and reduce thermal contact with tube walls. The lay of the screen is parallel to the direction of electric current flow. Two copper rods with diameter 0.25", welded directly to the copper strips on the heater, connect the heater to the power source.

The power source consists of two TCR-20T250 power supplies, each capable of producing 500 amps of current. The power supplies are load balanced and operate in parallel. The actual power supplied is dependent on the resistance of the nichrome grid, which changes with temperature. The power supplies are computer controlled using a software-implemented controller to stabilize the output power, although fluctuations on the order of ±1% do occur with frequency 60 Hz. Location of the heater can be easily changed within the tube.

The mean air flow through the Rijke tube is provided by a GAST R1102 blower, operating at 3450 rpm with a maximum throughput of 0.0127 m^3/s at standard atmospheric conditions. The blower is operated at full capacity with a 2" by-pass ball valve controlling the amount of air drawn through the damping chamber, or from the atmosphere. The flow rate is measured using a laminar flow element (Meriam 50MW20) and a differential pressure transducer (Honeywell Microswitch). This measurement takes place between the damping chamber and the blower. A thermocouple, located upstream of the laminar flow element, is used to correct for air density and viscosity to produce the total air mass flow rate.

Pressure transducers used in this experiment must be able to provide accurate measurements in a hot environment. The transducers used were PCB model 112A04, coupled with a 422D11 charge amplifier and a 482A20 signal conditioner. Charge-mode piezoelectric transducers were used, since the majority of the electronics is located in a separate charge amplifier, increasing the operating temperature range while retaining relatively high sensitivities. The two pressure transducers are flush mounted in the tube at positions x/L = 0.15 and x/L = 0.80.

In determination of stability boundaries, an array of 15 type K thermocouples is suspended from the top of the tube to the centerline. An additional thermocouple is located just before the laminar flow element that measures the mean flow through the tube. The spacing was selected to place more thermocouples nearer to the heat source, as well as to allow the heater to be located at key locations without interfering with the thermocouples. For validating the thermal modeling, some thermocouples were used at the exit and entrance cross-sections for determination of averaged temperature in those sections. Since the thermocouples have a relatively large time constant, they are multiplexed and sampled at 2 Hz. It is not possible for thermocouples to respond quickly enough at the acoustic time scales required in the experiment. They are used solely for time averaged temperature measurements.

In order to provide accurate measurements of the acoustic pressures and other relevant parameters in the Rijke tube, a fast sampling system is required. The data acquisition system is based on a Pentium III 700 MHz computer. A Computer Boards' CIO-DAS1602/12 (12 bit) data acquisition board is installed in the machine, using Sparrow control program as the software interface. An EXP-16 expansion board accommodates the 16 thermocouples in a multiplexed array and also provides cold junction compensation. The DAS1602/12 is operated in single-ended mode, giving a total of up to 16 analog input channels. It also contains two analog output channels, one of which is used to control the power supplies. In this configuration, data could be acquired in short bursts at over 8000 Hz, and for extended periods of time streaming to the hard drive at over 4000 Hz. For this Rijke tube, the frequencies of the primary excited modes are approximately 180 Hz and 360 Hz. These frequencies and the waveforms are easily captured by the data acquisition system.

6 A Précis of Results

6.1 Results from PLIF and Chemiluminescence Measurements

A variety of results have been forthcoming, ranging from the fundamental to the directly applicable. On the fundamental side, hysterisis was demonstrated and modeled in the electric Rijke tube, combustion and NO production responses were quantified for the jet-mixed burner, and a three-way coupling was found to be important in combustion instabilities. On the practical side, our results have made progress in defining what circumstances chemiluminescence is useful and when it is insufficient and OH PLIF is required for characterizing burner acoustic behavior. The experiments also showed the benefit of employing the laser absorption probe to directly measuring fuel mixing.

Figure 7 shows the chemiluminescence derived flame base location and amplitude of oscillation for two different configurations. When applied to burners that are to be operated in the same configuration as they are being tested, chemiluminescence is accurate in assessing flame oscillations and in guiding burner tuning. Slight modification of the jet mixed burner to shift the resonant frequency and reduce resonant frequency sensitivity are effective and can be measured with sufficient accuracy. The drawback of chemiluminescence is the inability of extrapolating these results to other burner or combustor arrangements.

Figure 8 compares chemiluminescence to OH PLIF data. It is immediately evident that the line-of-sight integration and associated image smearing inherent to chemiluminescence make detection of local oscillation impossible. OH PLIF, on the other hand, captures these fine scale oscillations without difficulty. This ability makes accurate calculation of Rayleigh index possible.

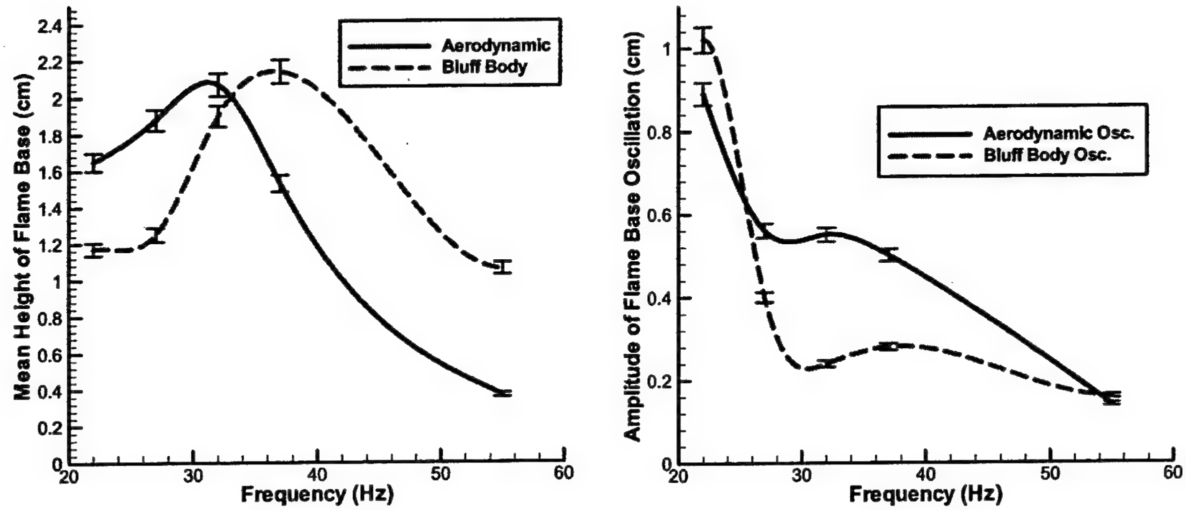

Figure 7. **Mean height of flame base (left) and amplitude of flame base oscillations (right) for both the aerodynamically and bluff-body stabilized burner configurations. The experimental values are at the center of each error bar and the curves are splines fitted to those values.**

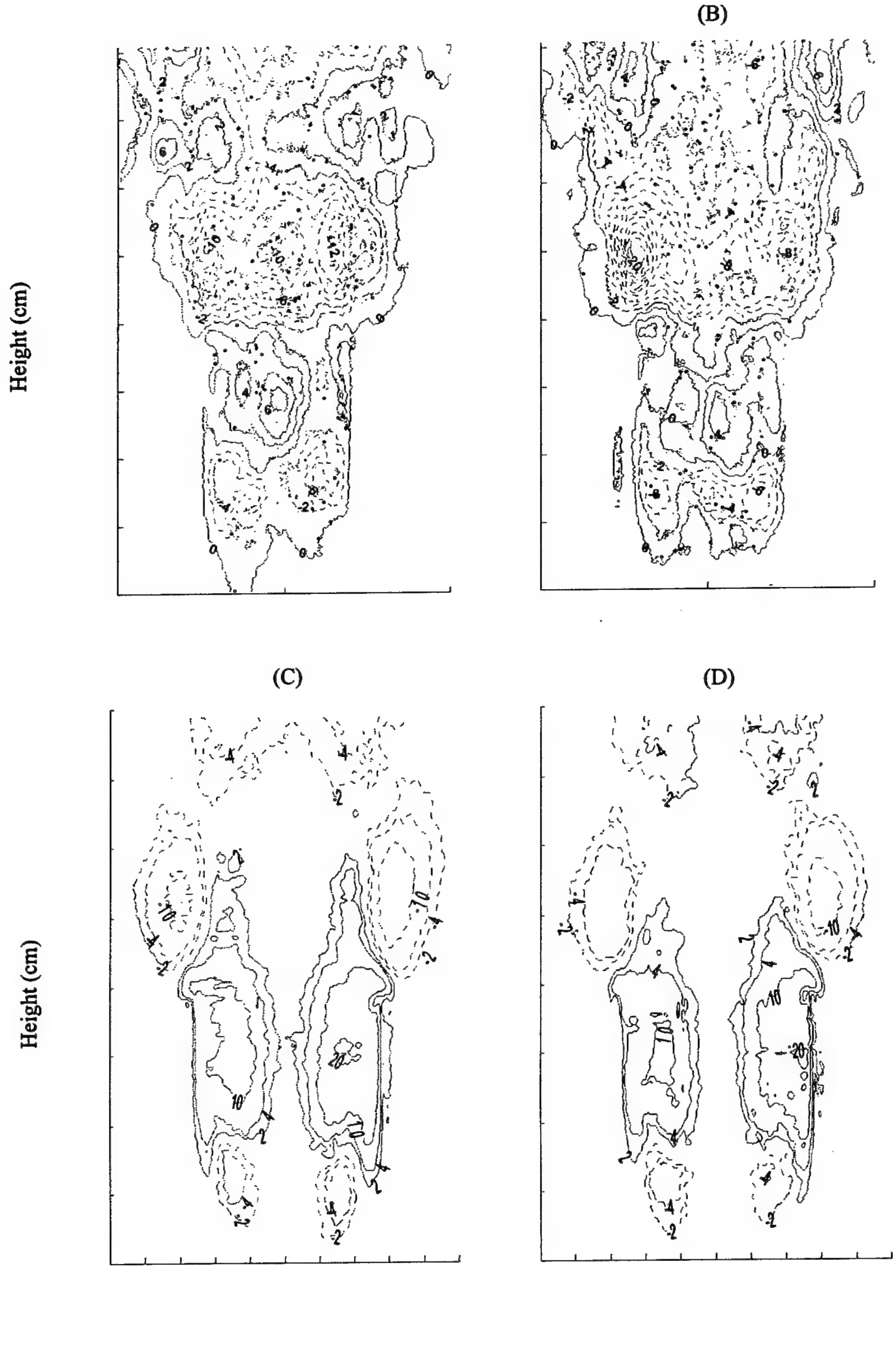

Figure 8. **Contour plot of Rayleigh Index for 32 Hz driving frequency: chemiluminescence of (A) the aerodynamically stabilized burner and (B) the bluff-body stabilized burner, OH PLIF of (C) the aerodynamically stabilized burner and (D) the bluff-body stabilized burner. Dashed contour lines indicate negative index values.**

Figure 9 shows plots of Rayleigh index and NO production response. It can be seen that while the plots are similar, NO production occurs at the high temperatures present in the flame front and then diffuses elsewhere. While allowing quantification, these plots in themselves do not elucidate the flame-acoustic wave coupling mechanism. Also, with such broad regions, it does not seem likely that weak pressure changes (order 1%) would results in much larger changes in heat release and NO production (order 10%). This type of magnifying change is more often the result of a system that has transitioned from stable to unstable (or at least marginally so). Since such transitions are not normally evident in this flame or with the present flow conditions, it is reasonable to suspect that a different type of flow stability has been affected.

This is the motivation for employing the Dibble-type probe to measure fuel concentration, and especially in the mixing layer of the jet. If the acoustic forcing couples to a mixing layer instability, then one could expect to see large mixture fraction oscillations and resulting flame oscillations. Figure 10 shows that acoustic forcing has no impact on the mean methane concentration. Figure 11 on the other hand shows that there is a very strong spike for mixture fraction oscillations in response to an imposed acoustic field. This spike is not only narrow in frequency, but also matches the driving frequency as the frequency was varied from 22 to 55 Hz. The spike was between 2 and 3 orders of magnitude stronger than the natural, low frequency oscillations that are present. This clearly demonstrates that there is a strong three-way coupling between flame oscillations, the resulting acoustic field, and that acoustic field driving oscillations in the fuel mixture fraction in the pre-flame zone.

Present work is focused on acquiring acetone PLIF images of these fuel oscillations to better characterize their structure and determine the type of instability mechanism responsible for the acoustic susceptibility.

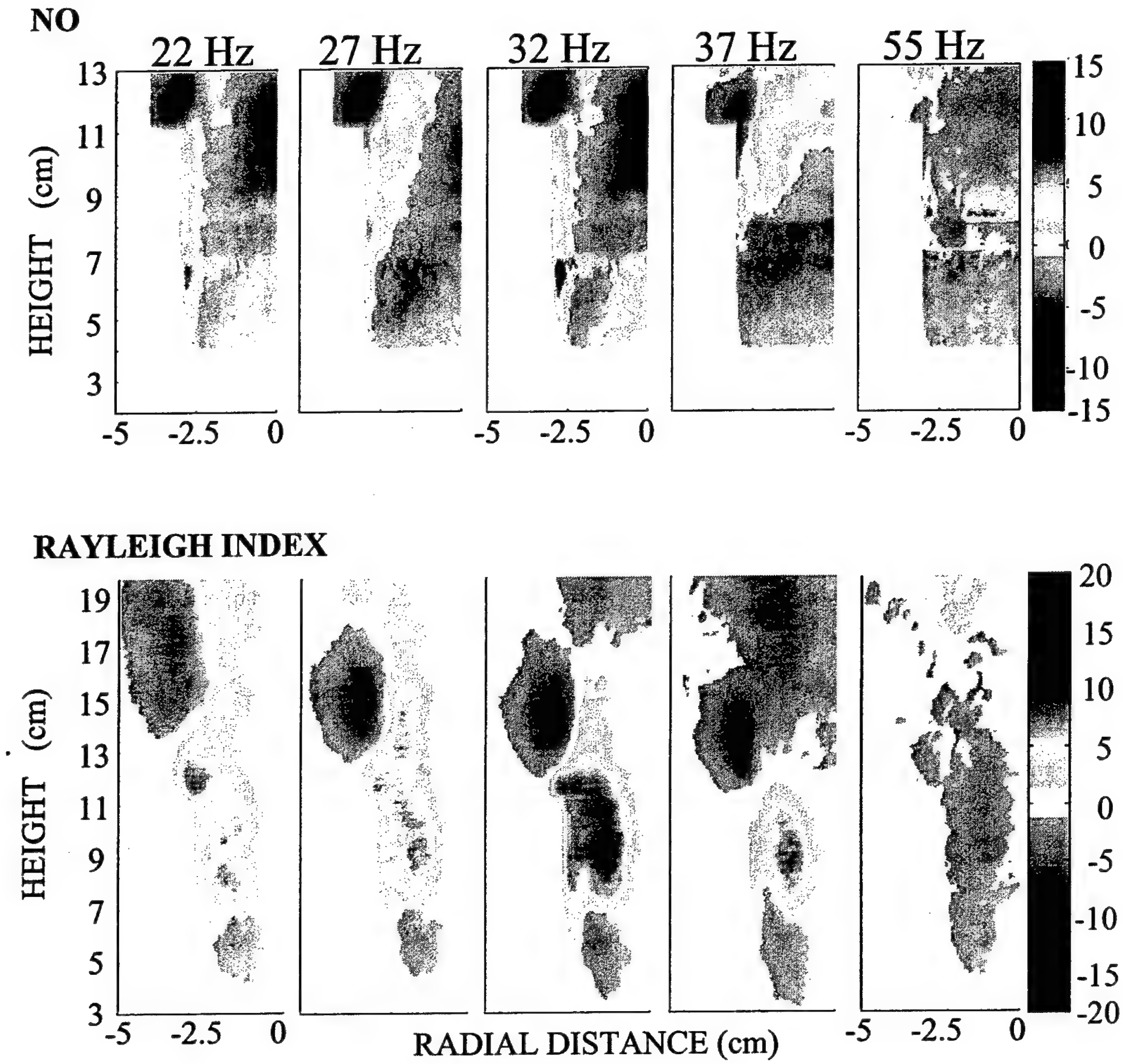

Figure 9. **NO Chem-Rayleigh (top) and Rayleigh index (bottom) for 22 to 55 Hz. Height is measured from the top of the eductor and the quartz enclosure ends at 11 cm. Only the left half of the images is shown, with the enclosure wall located at –3 cm.**

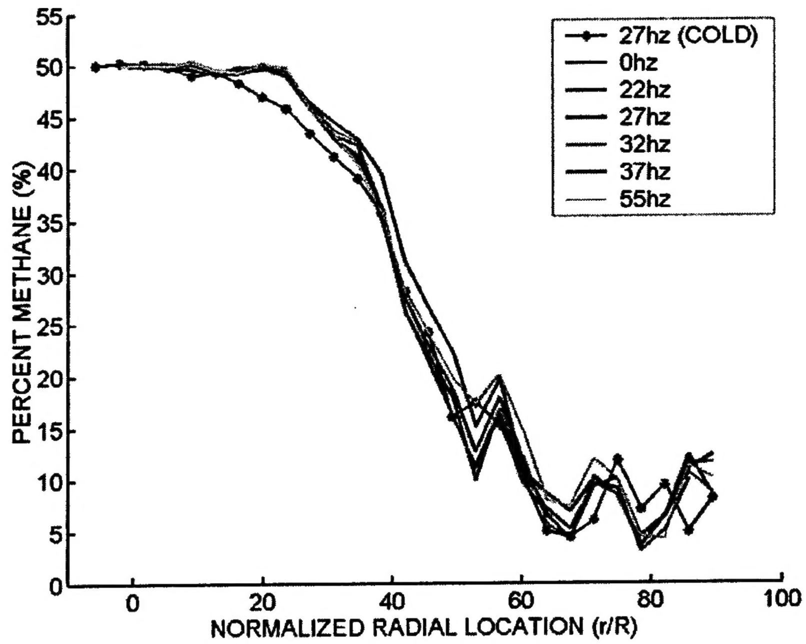

Figure 10. Methane concentration for reacting flows time-averaged at each radial location and for each driving frequency. The 27 Hz non-reacting case is shown for comparison.

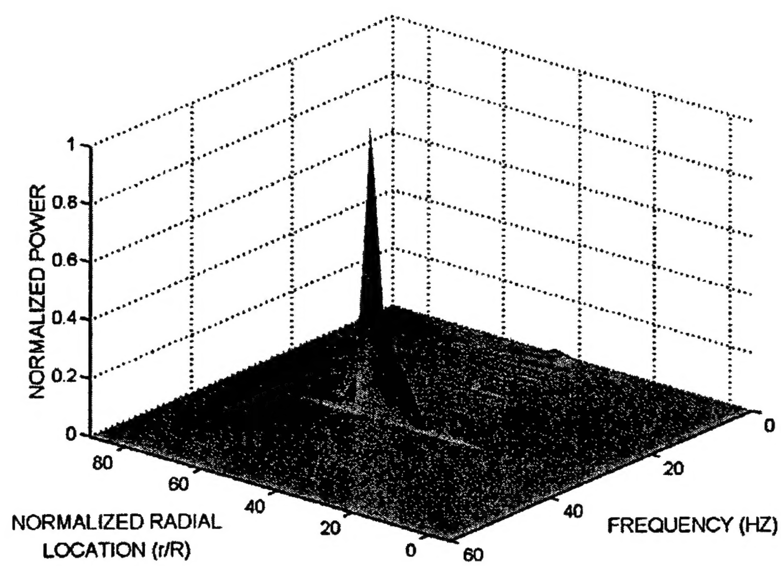

Figure 11. 32 Hz power density spectrum.

6.2 The Rijke Tube

Although the qualitative reasons for Rijke oscillations were understood prior to our project, there were uncertainties related to the unsteady heat transfer at the heater. Also, there was lack of both experiments providing data with specified measurement errors and models capable of accurately predicting the transition to instability and the properties of unstable regimes of thermoacoustic devices. Our work has been carried out with two primary intentions: to obtain accurate data for the stability boundary and the limit cycles of thermoacoustic oscillations in the Rijke tube, and to develop a theory that could predict the transition to instability and explain the nonlinear phenomena observed in the tests. The stability boundary has been quantified for three characteristic heater locations (1/4, 1/8, and 5/8 of the tube length) favorable for excitation of the lowest eigen acoustic modes of the pipe. An example of the stability boundary for one heater location and for increasing power variation is shown in Figure 12.

An unexpected experimental result was the observation of apparent hysteresis in the stability boundary in certain ranges of system parameters: the stability boundary is dependent on the direction of power variation at high mass flow rates at heater location $x/L = 1/4$, and at any flow rates at $x/L = 5/8$. That means that two stable states (either excited or with no oscillations) can exist at the same values of the main system parameters, and the choice of a particular state depends on the history of the experiment. The properties of the limit cycles have been quantified for the unstable domains of operation of the Rijke tube. Significant axial temperature variations along the pipe in both stable and excited regimes were measured during the tests.

To predict the onset of thermoacoustic oscillations a linear acoustic theory has been formulated. To obtain satisfactory theoretical results, it was essential to account for the axial variations of the temperature, and a thorough thermal analysis was carried out to determine the temperature field. The computed results are in quite good agreement with the stability boundaries observed.

A rather simple theory is described in this work for the nonlinear modeling of a complex thermoacoustic phenomenon in the presence of a mean flow. The results confirm the speculation that the nonlinear behavior of Rijke tubes of the sort investigated in this work is dominated by nonlinear characteristics of heat transfer from the source of the instabilities. A quasi-steady assumption for the mean heat transfer rate is invoked, and a hypothesis for the general form of

the heater transfer function that accounts for finite flow disturbances is developed. This theory demonstrates satisfactory agreement with experimental results for two locations of the heating element, corresponding to the different excitable modes of the Rijke tube. Discovered experimentally, the hysteresis effect in the transition between stable and excited states is explained theoretically Limit-cycle properties of higher harmonics of a self-excited mode can be estimated using a modal analysis and assuming uniform temperature field.

The results obtained in this work suggest that the approach developed should be applicable to assessing the stability of motion and the limit cycles in other thermal devices, such as combustion driven Rijke tubes, combustors, and thermoacoustic engines, if the Mach number of the average flow is small. Two generally important points must be emphasized: nonuniformities of the temperature field significantly affect the stability boundaries; as always in the case for unsteady behavior in combustion systems, the greatest unknown is the interaction (feedback) between the unsteady motion and the source of energy. The behavior of the Rijke tube observed and explained here clearly confirms these points.

6.3 Modeling of Combustion Instabilities in the Systems with Vortex Shedding

Another problem having a thermoacoustic origin and important for rocket engines and various burners has been studied in our program. Interactions between vortex shedding, combustion and chamber acoustics can lead to the appearance of large pressure oscillations. Better understanding of the complex nature of this phenomenon, which is a type of combustion instability, is achieved by applying the apparatus of dynamical systems to the simple reduced-order model offered here. Two novel ideas are introduced: the criterion for vortex shedding, based on the quasi-steady hypothesis, and an application of the kicked oscillator model for analysis of this type of combustion instability. The vortex shedding sub-model is validated against test data, producing the lock-in effect in vortex shedding as observed in experiments carried out elsewhere. The second idea, application of the kicked oscillator model, allows us to work out a fast and inexpensive calculation giving results that are easily interpreted. A limited comparison with available experimental results obtained at the Caltech dump combustor has been accomplished. An important factor for accurate modeling is the inclusion of a sufficient number of modes into the simulation. Recommendations are given for modal truncation in the

system considered. The model developed here can be further extended, including various additional physical effects. It is capable of simulating interesting nonlinear phenomena such as mode coupling, lock-in, hysteresis, and chaos, which are observed in real combustors.

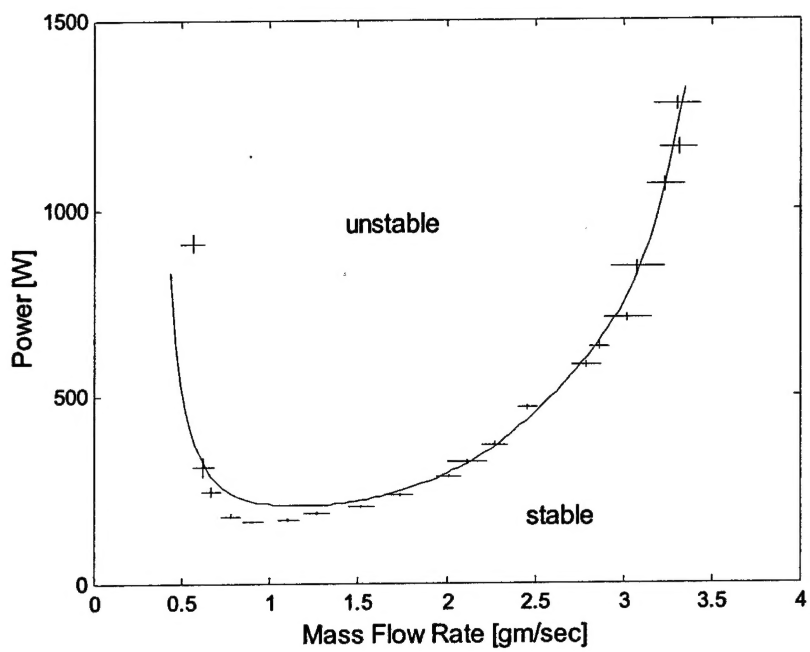

Figure 12: Comparison of the experimental (crosses) and calculated (solid line) results for transition to instability of the Rijke tube. Heater location x/L = 1/4.

7 Summary

This research effort has been widely successful: a number of papers published, students graduated, postdoctoral scholars trained, and insight achieved. Scientifically, the results have been a mix of the anticipated and the unexpected. It was anticipated that spatially resolved PLIF measurements would be more accurate than chemiluminescence for determining combustion response, as was demonstrated. Surprisingly, the chemiluminescence data was accurate in predicting trend-wise behavior in Rayleigh Index, but completely inaccurate in predicting whether the Index was positive or negative. Similarly, it was expected that the Dibble probe would show oscillations in the fuel mixture fraction. This was detected, but the oscillations turned out to be much stronger than expected and this has led to revisions in understanding of acoustically-induced mixing and its role in combustion instabilities.